Investigating Trinomial Tree Models for Option Pricing

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**Introduction**

Accurately pricing derivative securities, such as options, is a fundamental challenge in financial engineering. Among the various numerical methods developed for this purpose, lattice-based models like the binomial and trinomial trees stand out for their conceptual clarity and adaptability. The trinomial tree extends the binomial approach by allowing for three potential price movements at each time step—increasing, decreasing, or remaining unchanged—thereby offering enhanced flexibility and improved numerical stability. In this paper, we examine the formulation and implementation of the trinomial tree model for both European and American options, considering cases with and without dividend payments. We further evaluate the model's convergence behavior and computational demands to better understand its practical strengths and limitations.

**Trinomial Tree Framework**

A trinomial tree models the evolution of an asset’s price by allowing three potential outcomes at each time step: an upward move, a downward move, or no change. This provides greater flexibility and symmetry, leading to improved convergence when estimating option prices. At each node, the asset price is adjusted by factors such as u = e^(σ√(2Δt)), d = e^(−σ√(2Δt)), and m = 1, where σ represents volatility and Δt is the time step. Probabilities for each transition are determined using a risk-neutral framework, ensuring that the tree is arbitrage-free and consistent with market expectations.

**European and American Options Pricing**

The trinomial tree model is well-suited for pricing both European and American options. For European options, which can only be exercised at maturity, the valuation involves working backward from the option’s terminal payoffs. At each node, the option value is the expected value of its future outcomes, discounted at the risk-free rate using the model’s transition probabilities.

American options require consideration of early exercise at every node. The model evaluates the intrinsic value of exercising immediately versus holding the option. The higher of the two becomes the option’s value at that node. This feature, combined with the flexibility of the trinomial structure, allows for more accurate handling of American-style derivatives, especially under complex market conditions.

**Dividends and Model Adjustments**

Dividend payments influence option pricing by reducing the expected value of the underlying asset. The trinomial model can accommodate this by adjusting either the underlying price at ex-dividend dates or modifying the transition probabilities based on a continuous dividend yield. American call options, in particular, are sensitive to dividend payouts, as early exercise may become optimal just before the ex-dividend date. By incorporating these dividend effects, the trinomial model can provide realistic valuations for options on dividend-paying stocks.

**Computational Complexity and Convergence**

Trinomial trees are computationally more intensive than binomial trees, as each node leads to three child nodes instead of two. The total number of nodes grows quadratically with the number of time steps, leading to increased memory and time requirements. However, the added computational cost is offset by better convergence behavior. Trinomial trees tend to converge faster to theoretical option values, making them preferable for precise pricing tasks, especially in cases involving American-style options or complex payoffs.

**Numerical Implementation and Observations**

To see how well the trinomial tree works in practice, we built the model in Python and ran it on several types of options. For European call options, the results were really close to the Black-Scholes formula, even when we didn’t use that many time steps. When we tested American options, the model did a good job handling early exercise, especially when dividends were involved. For instance, American call options on stocks that paid dividends tended to be worth less and were often exercised earlier in the tree. Overall, the model gave accurate results, ran in a reasonable amount of time, and adapted well to different scenarios.

**Conclusion**

The trinomial tree model is a useful tool for pricing different types of options, including European calls and American options that involve dividends. Compared to the binomial tree, it gives more accurate results and handles things like early exercise and dividend payments more effectively. Even though it takes a bit more computing power, the extra effort usually pays off because it gets closer to the results of more complex continuous-time models. Overall, it’s a practical method that works well for both learning and real-world applications in finance.

**References**

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