**Investigating Trinomial Tree Models for Option Pricing**

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**Abstract**

This paper examines the implementation and performance of trinomial tree models for option pricing. We present a comprehensive framework for pricing both European and American options, with and without dividend considerations. Using real market data from popular stocks, we investigate the model's convergence behavior and computational complexity compared to traditional analytical solutions. Our implementation demonstrates that trinomial trees offer superior convergence properties for complex options while maintaining reasonable computational efficiency. The enhanced flexibility of the three-state framework proves particularly valuable for American-style options with early exercise features and dividend-paying underlyings.

**1. Introduction**

Accurately pricing derivative securities, such as options, is a fundamental challenge in financial engineering. Among the various numerical methods developed for this purpose, lattice-based models like the binomial and trinomial trees stand out for their conceptual clarity and adaptability. The trinomial tree extends the binomial approach by allowing for three potential price movements at each time step—increasing, decreasing, or remaining unchanged—thereby offering enhanced flexibility and improved numerical stability.

In this paper, we examine the formulation and implementation of the trinomial tree model for both European and American options, considering cases with and without dividend payments. We further evaluate the model's convergence behavior and computational demands to better understand its practical strengths and limitations. By incorporating real market data through the yfinance API, we ground our analysis in practical applications that reflect current market conditions.

**2. Trinomial Tree Framework**

A trinomial tree models the evolution of an asset's price by allowing three potential outcomes at each time: an upward move, a downward move, or no change. This provides greater flexibility and symmetry, leading to improved convergence when estimating option prices.

**2.1 Mathematical Formulation**

At each node in the trinomial tree, the asset price is adjusted by factors:

* Upward movement: A black and white text

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* Middle (no change): m = 1
* Downward movementA black and white math symbol

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Where sigma represents volatility and Delta t is the time step.

The risk-neutral probabilities for each transition are calculated as:

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Where r is the risk-free rate and q is the dividend yield. These probabilities ensure the tree is arbitrage-free and consistent with market expectations.

**2.2 Implementation**

Our implementation of the trinomial tree model is structured as a Python class that handles various option types and features:

A screen shot of a computer code

AI-generated content may be incorrect.This implementation provides a flexible framework for exploring different option scenarios while maintaining numerical stability.

**3. European and American Options Pricing**

The trinomial tree model is well-suited for pricing both European and American options. For European options, which can only be exercised at maturity, the valuation involves working backward from the option's terminal payoffs. At each node, the option value is the expected value of its future outcomes, discounted at the risk-free rate using the model's transition probabilities.

American options require consideration of early exercise at every node. The model evaluates the intrinsic value of exercising immediately versus holding the option. The higher of the two becomes the option's value at that node.

**3.1 Backward Induction Algorithm**

The core of our implementation is the backward induction algorithm, which prices options by working from expiration backward to the present:

A screenshot of a computer program

AI-generated content may be incorrect.This algorithm effectively handles both European and American options by appropriately considering the early exercise feature when necessary.

**4. Dividends and Model Adjustments**

Dividend payments influence option pricing by reducing the expected value of the underlying asset. The trinomial model can accommodate this by adjusting the transition probabilities based on a continuous dividend yield.

**4.1 Market Data Integration**

To evaluate the impact of dividends on option pricing, we integrate real market data using the yfinance library:

A computer screen shot of a program code

AI-generated content may be incorrect.By incorporating actual dividend yields, we can more accurately model the behavior of American call options, which may become optimal to exercise early just before ex-dividend dates.

**5. Computational Complexity and Convergence**

Trinomial trees are computationally more intensive than binomial trees, as each node leads to three child nodes instead of two. The total number of nodes grows quadratically with the number of time steps, leading to increased memory and time requirements. However, the added computational cost is offset by better convergence behavior.

**5.1 Convergence Analysis**

We analyze the convergence properties of our trinomial tree implementation by comparing it with the Black-Scholes formula for European options:

A screenshot of a computer code

AI-generated content may be incorrect.This method allows us to examine how the trinomial tree model's accuracy improves with increasing time steps and the associated computational cost.

**6. Numerical Experiments and Results**

We conducted a series of numerical experiments to evaluate the performance of our trinomial tree implementation across different option types and market conditions.

**6.1 Experiment Setup**

For our experiments, we used real market data from popular stocks with varying dividend yields:

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A screen shot of a computer code

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**6.2 Results and Analysis**

Our experiments reveal several key insights into trinomial tree models:

1. **Convergence Rate**: Trinomial trees converge faster to theoretical option value than binomial trees, especially for at-the-money options.
2. **Dividend Impact**: For American call options on dividend-paying stocks, we observed significant price differences compared to their European counterparts, confirming the importance of early exercise consideration.
3. **Computational Cost**: While more computationally intensive than binomial trees, the trinomial model's improved accuracy justifies the additional cost for complex options.
4. **Market Data Integration**: Using real volatility and dividend data from yfinance resulted in more accurate option prices that reflect current market conditions.

The following visualization shows the convergence behavior for an at-the-money Apple call option:

A graph of a graph of a graph

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A graph with red lines

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A graph with red and blue dots

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**7. Conclusion**

The trinomial tree model provides a flexible and accurate approach for pricing various types of options. Our implementation and analysis demonstrate that it effectively handles both European and American options, with appropriate consideration for dividend effects.

Compared to the binomial tree, the trinomial model offers superior convergence properties and more accurate results, particularly for American options with early exercise features. While computationally more intensive, the improved accuracy justifies the additional computational cost, especially for complex options and real-world applications.

The integration of market data through yfinance enhances the model's practical relevance, allowing for more realistic option pricing that accounts for current market conditions and dividend policies. This approach bridges the gap between theoretical models and practical financial applications.

In summary, the trinomial tree model represents a valuable tool in the option pricing toolkit, offering a balance between computational efficiency and accuracy that makes it suitable for both educational and professional applications in financial engineering.

**References**

Boyle, P. P. (1986). Option Valuation Using a Three-Jump Process. *International Options Journal*.

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option Pricing: A Simplified Approach. *Journal of Financial Economics*.

Hull, J. C. (2018). *Options, Futures, and Other Derivatives*. 10th Edition. Pearson.

Derman, E., & Kani, I. (1994). Riding on a smile. *Risk*, 7(2), 32-39.

Figlewski, S., & Gao, B. (1999). The adaptive mesh model: a new approach to efficient option pricing. *Journal of Financial Economics*, 53(3), 313-351.